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ADDITIONAL REMARKS ON ATTRITION AND FEBA MOVEMENT COMPUTATIONS --ETC(U)
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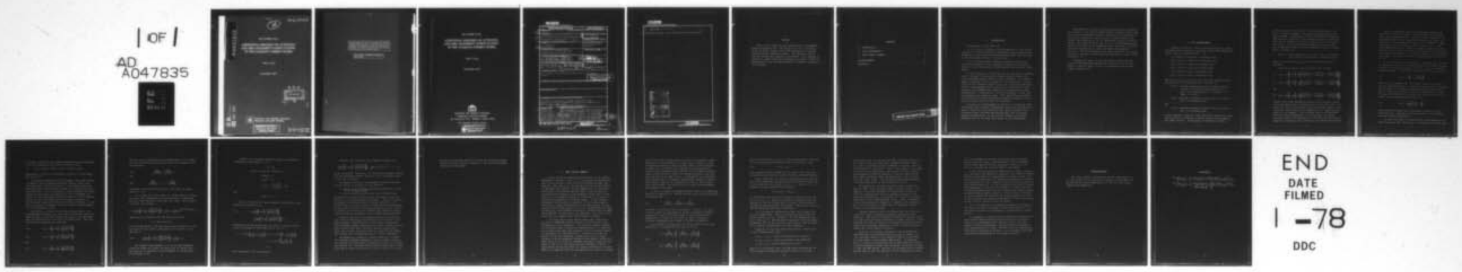

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IDA PAPER P-1277

ADDITIONAL REMARKS ON ATTRITION AND FEBA MOVEMENT COMPUTATIONS IN THE LULEJIAN-I COMBAT MODEL

Alan F. Karr

November 1977

INSTITUTE FOR DEFENSE ANALYSES
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PROGRAM ANALYSIS DIVISION
400 Army-Navy Drive, Arlington, Virginia 22202

IDA Independent Research Program

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER (14) P-1277	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) Additional Remarks on Attrition and FEBA Movement Computations in the Lulejian-I Combat Model.		5. TYPE OF REPORT & PERIOD COVERED (9) Final rept.,
		6. PERFORMING ORG. REPORT NUMBER IDA P-1277
7. AUTHOR(s) (10) Alan F./Karr		8. CONTRACT OR GRANT NUMBER(s) IDA Independent Research
9. PERFORMING ORGANIZATION NAME AND ADDRESS Institute for Defense Analyses Program Analysis Division 400 Army-Navy Drive, Arlington, VA 22202		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS (18) IDA/HQ, SBIE		12. REPORT DATE (11) Nov [redacted] 77
		13. NUMBER OF PAGES 28 (12) 23p.
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (19) 77-19529, AD-E500 003		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE NA

16. DISTRIBUTION STATEMENT (of this Report)

This document is unclassified and suitable for public release.

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Lulejian-I Combat Model, combat simulation model, theater-level combat simulation, attrition computation, FEBA movement computation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper discusses mathematical, physical, and logical properties of the attrition/FEBA movement calculations in the Lulejian-I theater-level combat model. An erroneous counterexample in ~~the paper~~ the writer's previous paper, *On the Lulejian-I Combat Model* (AD-A030 171), ~~by A.F. Karr~~ is acknowledged and a correct counterexample demonstrating nonexistence of solutions to the attrition/FEBA movement equations is given. Difficulties with logical cont on P1473B

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and physical bases of the computation are discussed.

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PREFACE

Work on this paper has been supported by the Independent Research Program of IDA. The paper is one of a continuing series of papers treating the current generation of theater-level ground-air simulation models. The papers are mostly critical reviews and comments on the models. Also underway is research on combat attrition processes, the nature of which is interdependent with the identification of important problems in the critiques.

CONTENTS

1. INTRODUCTION

The purposes of this paper are

(1) to correct an erroneous "counterexample" in the author's critique [2] of the Lulejian-I theater-level combat model, which purported to show that the attrition/FEBA movement equations used in the model need not have a solution; and

(2) to provide further comments concerning logical and plausibility aspects of the methodology used in the Lulejian-I model for joint calculation of FEBA movement and principal combat losses.

It was asserted in [2] that the set of simultaneous equations used in Lulejian-I to represent relations among maneuver unit losses and FEBA movement need not have a solution. The counterexample presented in [2] in support of this assertion is incorrect, but the assertion itself is true, as is demonstrated by the correct counterexample given in Section 2 of this paper. Incidentally, the statement and example in [2] concerning possible nonuniqueness of solutions are, strictly speaking, correct, but are irrelevant in practical terms. The computational structure of the model would leave unspecified separation distances at the previous values, which is entirely reasonable.

Although the counterexample in Section 2 is artificial both structurally and in terms of parameter values, and although it is easy to conjecture that solutions exist and are unique in all situations where the model has been applied, this conjecture remains unverified and, in the author's opinion, the prudent potential user of the model must regard it with skepticism.

Independent of questions involving existence and uniqueness of solutions to the attrition/FEBA movement equations (which many will view as inessential technical objections) are questions involving the logical and physical bases of these equations and whether they represent at all the underlying physical process of combat. The author believes that it can be convincingly argued that the equations do not constitute even a plausible (let alone realistic) description of combat and makes this argument in Section 3. While the argument is in part subjective, it is both cogent and coherent and the potential model user at least should be aware of it.

Although this paper is self-contained in the sense that notations and necessary equations are presented in full, the reader is urged to study it only in conjunction with the more complete analysis [2].

2. THE COUNTEREXAMPLE

Consider a bilateral combat between attacking and defending sides, each of which has infantry, tanks, and armored personnel carriers (hereafter APCs). Let

I_D = number of infantry on defending side,

ΔI_D = losses of infantry on defending side,

T_D = number of tanks on defending side,

ΔT_D = losses of tanks on defending side,

P_D = number of APCs on defending side,

ΔP_D = losses of APCs on defending side,

and let I_A, ΔI_A, T_A, ΔT_A, P_A, ΔP_A denote analogously defined quantities for the attacking side. Further, let

$R(U,V)$ = average separation distance between elements of type U on defending side and elements of type V on attacking side (I = infantry, T = tanks, P = APCs),

$c_D(U)$ = maximum acceptable attrition rate for units of type U on defending side,

and

$c_A(U)$ = maximum acceptable attrition rate for units of type U on attacking side.

In the Lulejian-I model all these quantities, except for input initial values, are dynamic variables whose values change as the combat evolves. For example, the maximum acceptable

attrition rates are functions of the surviving faction of a unit's original strength, even though replacements may have been received. Nonetheless, from the standpoint of attrition/FEBA movement computation, the "maximum acceptable" attrition rates may be treated as constants, which we do in the discussion below. Also, the attrition equations involve various kill potentials (details of their computation are not relevant here), so simply let

$k_D(U,V)$ = kill potential of one type U unit on defending side against type V units on attacking side,

and let kill potentials $k_A(U,V)$ for the attacking side be analogous.

Then the attrition equations become the following:

$$(1)\quad \Delta I_D = I_D\left(1-\exp\left[-\frac{1}{I_D}\left(\frac{k_A(I,I)I_A}{R(I,I)} + \frac{k_A(T,I)T_A}{R(I,T)} + \frac{k_A(P,I)P_A}{R(I,P)}\right)\right]\right)$$

$$(2)\quad \Delta T_D = T_D\left(1-\exp\left[-\frac{1}{T_D}\left(\frac{k_A(I,T)I_A}{R(T,I)} + \frac{k_A(T,T)T_A}{R(T,T)} + \frac{k_A(P,T)P_A}{R(T,P)}\right)\right]\right)$$

and

$$(3)\quad \Delta P_D = P_D\left(1-\exp\left[-\frac{1}{P_D}\left(\frac{k_A(I,P)I_A}{R(P,I)} + \frac{k_A(T,P)T_A}{R(P,T)} + \frac{k_A(P,P)P_A}{R(P,P)}\right)\right]\right),$$

together with three analogous equations that represent losses to the attacking side. We should note at this point that (1-3) above omit certain details and modifications that appear in the Lulejian-I model; however the omissions do not change the substance of our argument. To the extent that we can determine from the model documentation, (1-3) represent the correct functional form of the dependence of attrition calculations on numbers of resources, kill potentials, and separation distances.

Were the separation distances previously computed, the equations (1-3) would suffice to define attrition to various units on the defending side. However, a basic premise of the Lulejian-I model is that separation distances and losses are functions of one another; the purpose is to represent a trade-off between losses of resources and territory lost. That such trade-offs occur in combat, at least in some form, seems certain, but the extent to which the Lulejian-I model represents them appears questionable. For further comments on this problem the reader is referred to Section 3 below and also to Section 7 of [2].

It is therefore further assumed in the model that movement of each type of unit is a linear function of the ratio of actual attrition to maximum acceptable attrition. Thus, for example, forward movement of infantry on the attacking side is given by

$$M_A(I) = m\left(1 - \frac{\Delta I_A}{c_A(I) I_A}\right), \tag{4}$$

where m is the maximum rate of movement, which is achieved only in unopposed advance. Observe that since losses ΔI_A depend on the matrix R of separation distances, so does the movement $M_A(I)$. Analogous equations represent movements $M_A(T)$ and $M_A(P)$ of tanks and APCs on the attacking side, while movement of infantry on the defending side is given by

$$M_D(I) = m\left(\frac{\Delta I_D}{c_D(I) I_D} - 1\right), \tag{5}$$

where positive movement still represents an advance of the attacking side. Movements of tanks and APCs on the defending side are obtained analogously.

In order that there be a well-defined "FEBA movement," the model postulates that the matrix R of separation distances must

be chosen so that all six movements defined above are identical, i.e., the distances $R(U,V)$, must be chosen so that

$$(6) \qquad M_A(I) = M_A(T) = M_A(P) = M_D(I) = M_D(T) = M_D(P) .$$

Implications of (4-6) are discussed in Section 3 of this paper and also in [2].

The problem therefore is the following: with given values of resource levels, maximum acceptable casualty levels, and kill potentials, choose R so that (6) is satisfied, where the quantities involved are computed using (1-5) and omitted analogous equations. Since five properly chosen separation distances define the other four, the equation (6) can be regarded as a system of five simultaneous nonlinear equations in five unknowns. Especially since the equations are nonlinear, questions of existence and uniqueness of solutions cannot be ignored. It was with the problem of existence that the erroneous counterexample in [2] was supposed to deal.

Specifically, consider the following special case: the defending side possesses only tanks and the attacking side only tanks and APCs. We further assume that tanks on the defending side are invulnerable to APCs on the attacking side. The relevant attrition equations are therefore

$$(7a) \qquad \Delta T_A = T_A\left(1-\exp\left[-\frac{k_D(T,T)T_D}{T_A R(T,T)}\right]\right),$$

$$(7b) \qquad \Delta P_A = P_A\left(1-\exp\left[-\frac{k_D(T,P)T_D}{P_A R(T,P)}\right]\right),$$

and

$$(7c) \qquad \Delta T_D = T_D\left(1-\exp\left[-\frac{k_A(T,T)T_A}{T_D R(T,T)}\right]\right).$$

Without loss of generality we may assume that $m = 1$, in which case the movement equations and the system (6) reduce to the two equations

(8a) $$\frac{\Delta T_A}{c_A(T)T_A} = \frac{\Delta P_A}{c_A(P)P_A} \quad ,$$

and

(8b) $$\frac{\Delta T_D}{c_D(T)T_D} - 1 = 1 - \frac{\Delta T_A}{c_A(T)T_A} \quad .$$

One wants to choose $R(T,T)$ and $R(T,P)$ such that (8) holds, subject to (7).

It is shown in [2] that there is a unique value of $R(T,T)$ such that (8b), which does not involve $R(T,P)$, is satisfied. One must then find $R(T,P)$ such that (8a) holds. After simplifications, (8a) becomes

(9) $$\frac{1}{c_A(T)}\left(1-\exp\left[-\frac{k_D(T,T)T_D}{T_A R(T,T)}\right]\right) - \frac{1}{c_A(P)} + \frac{1}{c_A(P)}\, a^{k_D(T,P)T_D/P_A} = 0 \ ,$$

where $R(T,T)$ is fixed by (8b) and where we have put

$$a = \exp[-1/R(T,P)] \ .$$

It is further shown in [2] that there exists a solution to (9) that lies in $(0,1)$, which corresponds to $0 < R(T,P) < \infty$, if and only if

(10) $$\frac{1}{c_A(T)}\left(1-\exp\left[-\frac{k_D(T,T)T_D}{T_A R(T,T)}\right]\right) - \frac{1}{c_A(P)} \leq 0 \ .$$

The alleged counterexample in [2, p.50] gave parameter values for which (10) fails, but which fail to satisfy (8b) and hence do not represent a counterexample to the existence of solutions to (8).

However, the following parameter values do constitute a legitimate counterexample. Let

$$T_A = T_D$$

$$k_D(T,T) = k_D(T,P) = k_A(T,T) = 1$$

$$R(T,T) = 1$$

$$c_A(T) = .6$$

$$c_D(T) = \frac{3 - 3e^{-1}}{1 + 5e^{-1}} \sim .657$$

and

$$c_A(P) = 1 .$$

First, we verify that these parameters satisfy (8b); that equation can be written as

$$(11) \qquad 2 - \frac{1}{c_A(T)}\left(1\text{-}\exp\left[-\frac{k_D(T,T)T_D}{T_A R(T,T)}\right]\right) - \frac{1}{c_D(T)}\left(1\text{-}\exp\left[-\frac{k_A(T,T)T_A}{T_D R(T,T)}\right]\right) = 0 .$$

Substitution of the values above for $k_A(T,T)$, $k_D(T,T)$, $R(T,T)$, T_A and T_D changes the left-hand side of (11) to

$$\begin{aligned} 2 - (1\text{-}e^{-1})\left[\frac{1}{c_A(T)} + \frac{1}{c_D(T)}\right] &= 2 - (1\text{-}e^{-1})\left[\frac{5}{3} + \frac{1 + 5e^{-1}}{3 - 3e^{-1}}\right] \\ &= 2 - (1\text{-}e^{-1})\left[\frac{18}{9(1\text{-}e^{-1})}\right] \\ &= 0 \end{aligned}$$

and consequently (8b) is satisfied.

However, (10) fails for these parameter values since

$$\frac{1}{c_A(T)}\left(1\text{-exp}\left[-\frac{k_D(T,T)T_D}{T_A R(T,T)}\right]\right) - \frac{1}{c_A(P)} = \frac{5}{3}\,(1\text{-}e^{-1}) - 1 \sim .05\ ,$$

which is positive. Therefore, for these given parameter values, the attrition/FEBA movement equations of the Lulejian-I model do not have a solution.

We emphasize that our counterexample, although mathematically correct, is artifical in two respects:

(1) Tanks on the defending side are invulnerable to APCs on the attacking side;

(2) The acceptable casualty rates are extremely high.

The author's intuitive belief is that the former is more important to the counterexample given above. Indeed, a plausible conjecture is that there exists a unique solution to (6) provided that no class of units on either side be invulnerable to any class of units on the other and that all types of units be present on each side, both of which are reasonable assumptions in terms of the physics of the combat being represented.

However, and this is an important point, the burden now rests with the developers and proponents of the model to provide a rigorous identification of those cases in which the attrition/FEBA movement equations do not have a unique solution, since we have shown that a solution need not *always* exist. Until this is done, we believe that the model must be viewed with a great deal of skepticism and very little confidence.

Incidentally, it would be a rather simple job to modify the Lulejian-I computer programs to check by simple substitution whether the iteratively calculated solution to (6) in fact does satisfy (6). The user who insisted on employing the model could then at least be alerted if a solution were not obtained.

We feel that possible difficulties with the iteration scheme itself are minor, however, compared to the other shortcomings discussed in this paper.

3. SOME FURTHER COMMENTS

In this Section we amplify and extend some of the comments appearing in Sections 7 and 8 of [2] concerning the methodology used in the Lulejian-I model for attrition/FEBA movement computation. The tone of [2] and possibly also the second section of this paper may indicate to some readers that our principal criticisms of this methodology are based on the existence of artificial and unrealistic counterexamples rather than (in the opinion of such readers) grounds of substance in the context of computerized simulation of military combat. On the contrary, although we believe that the existence of any counterexample constitutes a significant weakness of the methodology, we also believe several other aspects of it to be considerably less than satisfactory, even assuming there were no problems with existence and uniqueness of solutions to the equations. It is these criticisms that the comments below are intended to present.

1) No doubt there occurs during combat some trade-off on the part of commanders between losses of territory and losses of resources (i.e., casualties). We question, however, whether the Lulejian-I model contains any representation of a trade-off in the usual sense that some quantity is to be optimized subject to constraints on other quantities. It seems to us that the system (6) does not represent a trade-off at all but simply attempts to choose separation distances that equate, within each side, the ratio of actual to "maximum acceptable" losses for all types of units (cf. (12) below) and also attempts to relate these ratios for the two sides in a manner discussed in 2) below. There is no physical basis for assuming that all units on a given

side should incur the same ratio of actual to acceptable losses; indeed our intuition (which, of course, may be faulty) is that an important reason for having several types of units available in combat is the flexibility attainable by allowing them to sustain (over relatively short periods of time) rather different rates of attrition even though the maximum acceptable long term loss rate may be the same for all. Consequently, the Lulejian-I model appears not to represent a significant property of diversified forces; indeed it is constructed so that it *cannot* represent this property. We believe this is a major and unalterable shortcoming of the model.

2) Let us consider in somewhat greater detail the assumptions implicit in equation (6). Within one side, say the attacking side, (6) implies that

$$\frac{\Delta I_A}{c_A(I)I_A} = \frac{\Delta T_A}{c_A(T)T_A} = \frac{\Delta P_A}{c_A(P)P_A}, \tag{12}$$

the effect of which has been discussed in 1) above: all types of units suffer the same ratio of actual to maximum acceptable casualties. Even if different types of units had different maximum rates of advance, (6) would not admit a physically plausible foundation, even though (12) would be modified.

The relation of these ratios on the two sides is even less defensible. It follows from (6) that if we put

$$\ell_A = \frac{\Delta I_A}{c_A(I)I_A} \left(= \frac{\Delta T_A}{c_A(T)T_A} = \frac{\Delta P_A}{c_A(P)P_A} \right)$$

and

$$\ell_D = \frac{\Delta I_D}{c_D(I)I_D} \left(= \frac{\Delta T_D}{c_D(T)T_D} = \frac{\Delta P_D}{c_D(P)P_D} \right)$$

which are the ratios of actual to maximum acceptable casualties for the attacking and defending sides, respectively, then

$$(13) \qquad 1 - \ell_A = \ell_D - 1 \ .$$

This assumed relation between the two sides' loss rates has no physical basis and also several rather strange implications.

First, note that (13) cannot be satisfied unless exactly one of the quantities ℓ_A and ℓ_D exceeds one and the other lies in the interval (0,1), unless $\ell_A = \ell_D = 1$. However since by equation (5) FEBA movement M is given by

$$M = m(1-\ell_A) = m(\ell_D-1) \ ,$$

in order that M not exceed m in absolute value, one must have $\ell_A \leq 2$ and $\ell_D \leq 2$. There is, however, no assurance that the system (6), even if it does have a solution, has a solution satisfying these additional constraints. Hence there are still further unresolved existence difficulties.

It is the physical difficulties with (13), though, that are most significant. There is no physical basis, since in general either ℓ_A or ℓ_D exceeds 1, for the assumptions that $\ell_A \leq 2$ and $\ell_D \leq 2$. Why can actual casualties exceed acceptable casualties, but not by a factor of more than 2? We can think of no physical justification for this assumption.

Equation (13) has even worse implications. Exactly one of the following must occur:

a) $\ell_A < 1$, $\ell_D > 1$ and the attacking side advances;

b) $\ell_A > 1$, $\ell_D < 1$ and the attacking side withdraws;

c) $\ell_A = \ell_D = 1$ and the FEBA does not move.

While it is unrealistic that the FEBA cannot move unless one side exceeds its acceptable casualty rate, it is utterly

unbelievable that the only way the FEBA can *fail* to move is if both sides suffer precisely their maximum acceptable levels (in terms of surviving initial strengths) of losses. This is unbelievable for two reasons: first, there seems to be a range of force levels, capabilities, and losses over which neither side generates sufficient momentum to move the FEBA; second, many combat situations where the FEBA fails to move involve only very light losses. The Lulejian-I model seems incapable of representing either of these important effects.

We should point out that there are other circumstances when the FEBA in the Lulejian-I model can fail to move. For example, on the day following an "unsuccessful" attack, in which the attacking side fails to advance according to the attrition calculations, the situation in a sector is declared "semiactive" and no FEBA movement occurs. This is somewhat, but not unquestionably, reasonable. However, this semiactive state can be entered, it seems, only if condition c) above obtains on the preceding day.

Observe, moreover, that (13) implies that the side that incurs more than its maximum acceptable losses also loses territory. This means that whatever "trade-off" between territory and losses the model represents is purely negative: a side withdraws in order to reduce its casualties and cannot sustain higher than acceptable casualties in order to gain territory (which may clearly happen in reality).

Finally, (13) imposes a symmetry on the attacking and defending sides that probably does not exist in combat: by (13) each side reacts, in the Lulejian-I model, in precisely the same manner to unacceptably high losses. As mentioned above, in reality the attacking side in particular may choose to incur momentarily higher than acceptable casualties in order to keep the FEBA moving. Further, the defending side in reality may not seek to move the FEBA even if its casualties are very

low; it may simply be content with maintaining its current position and low losses. Representation of these phenomena is not simply omitted from the Lulejian-I model--it is explicitly excluded and cannot be added to the model.

3) The inverse exponential dependence of losses on separation distances that appears in equations (1-3) above is without physical justification or plausibility and seems to have been chosen only because nearly all attrition equations in the model are exponential in form. Research efforts by the developers of the Vector-I model (cf. [1] for a review by the author) indicate that dependence of attrition on distance is a subtle and complicated phenomenom that, even in a theater-level model, cannot be represented by simple equations. While it may be possible to derive alternative equations, we question whether, in view of the conceptual and physical inadequacies of the entire methodology, to do so would be worth the effort.

4) In the same way, the assumed dependence of movement on the ratio of actual to acceptable losses that appears in (5) has neither physical nor mathematical justification or plausibility.

To summarize, we believe that the assumptions inherent in the attrition/FEBA computations appearing in the Lulejian-I model are so incompatible with physical reality that they render the model significantly and unpredictably inaccurate as a representation of combat. Together with the mathematical difficulties discussed in Section 2, the criticisms given here constitute strong grounds for not using the model.

ACKNOWLEDGEMENT

The author wishes to express his sincere appreciation to Dr. Royce Kneece of the Land Forces Division of OASD(PA&E) for discovering the errors appearing in [2] and for many helpful comments and suggestions.

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[1] Karr, A.F. *On the Vector-I Combat Model.* P-1181. Arlington, VA.: Institute for Defense Analyses, 1976.

[2] Karr, A.F. *On the Lulejian-I Combat Model.* P-1182. Arlington, VA.: Institute for Defense Analyses, 1976. (AD-A Ø3Ø 171)